B-math 2nd year Back paper Exam
Subject : Analysis III

Time : 3.00 hours Max.Marks 65.

1. Let S be a parametric surface described by the explicit formula $z = f(x, y)$, where $(x, y)$ varies over a plane region $T$, the projection of S in the $xy$ plane. Let $\vec{F}(x, y, z) := P\ \vec{i} + Q\ \vec{j} + R\ \vec{k}$ where $P, Q, R$ are functions on $S$ and let $\vec{n}$ be the unit normal to $S$ having a non negative $z$ component. Show that

$$\iint_S \vec{F} \cdot \vec{n}\ dA = \iint_T (-P\partial_x f - Q\partial_y + R)\ dxdy.$$

(10)

2. Let $\vec{F} := y^2\vec{i} + xy\vec{j} + zx\vec{k}, (x, y, z) \in \mathbb{R}^3$. Let $S$ be the surface in $\mathbb{R}^3$ (hemisphere) given by $x^2 + y^2 + z^2 = 1, z \geq 0$. Evaluate $\iint_S \vec{F} \cdot \vec{n}\ dA$. (10)

3. Show that the moment of inertia of a homogenous thin spherical shell $S$ about a diameter is equal to $\frac{2}{3}ma^2$, where $m$ is the mass of the shell and $a$ is its radius. Recall that the moment of inertial $I_L$ of an object about a line $L$ is $I_L := \iint_S \delta(x, y, z)^2 f(x, y, z)\ dA$ where $\delta(x, y, z)$ is the distance of the point $(x, y, z)$ from the line $L$ and $f(x, y, z)$ is the density at $(x, y, z)$. (10)

4. a) Let $\{f_n\}$ be a sequence of continuous functions on $[0, 1]$ such that $f_n \to f$ uniformly on $(0, 1)$. Show that $f$ is uniformly continuous on $(0, 1)$.
b) Let $f_n(x) := \frac{sin(nx)}{n}$ $x \in [0, 1]$. Show that $\lim\limits_{n\to\infty} \int_0^1 f_n(x)dx = 0$.
c) Answer the same question as in b) for the sequence $f_n(x) := \frac{1}{nx+1}$ $x \in [0, 1]$.
(5+5+5)

5. Let $f : \mathbb{R}^2 \to \mathbb{R}^2, f(x, y) := (x^2 - y^2, 2xy)$.
a) Show that $f$ is one to one on $A := \{(x, y) : x > 0, y \in \mathbb{R}\}$.
b) Describe the set $f(A)$.
c) Find $D(f^{-1})(0, 1)$. (10)

6. Let $Q \subset \mathbb{R}^2$ be a rectangle. Let $f : Q \to \mathbb{R}$, be bounded and continuous. Show that $f$ is Reimann integrable on $Q$. (10)